



LC2 Calculation Policy

Progression in Calculations

Introduction

This policy was created in response to a need across Learning Community 2 schools and after discussion between Headteachers; to have a standard calculation policy that met the National Curriculum guidelines and provided a consistent approach to calculation that could be used within year groups across LC2 schools and as a progression through the phases.

The policy has been developed with a concrete, pictorial and abstract (CPA) approach in order to scaffold and develop pupils' understanding. The CPA approach is relevant across all ages in order to secure understanding and gain conceptual and procedural fluency.

If children are not at age related expectations for their year the policy will be helpful in tracking back to previous learning. This will enable pupils to develop understanding of calculation methods rather than to have to memorise procedures.

We have given an indication of pitch for the stages in our calculation policy.

A note about carrying digits – the examples in this document reflect the variety of carrying methods used across schools.

NB: This is version 2 of the LC2 Calculation Policy and incorporates changes made during the Maths Subject Leader Session on Thursday 19th October 2017.



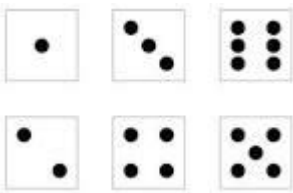

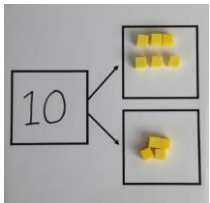

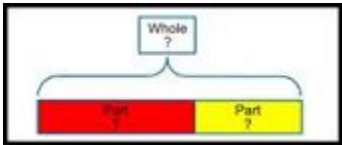
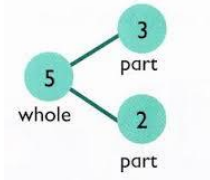

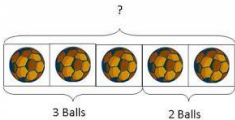

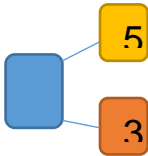
LC2 Calculation Policy


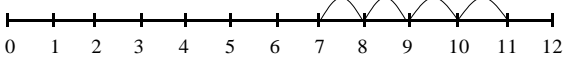
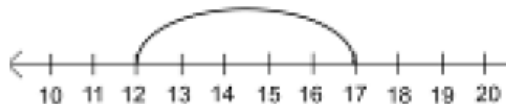
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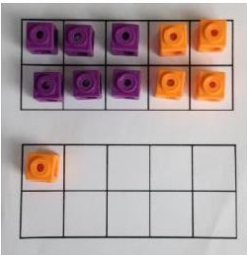
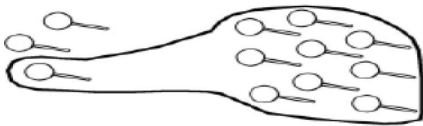
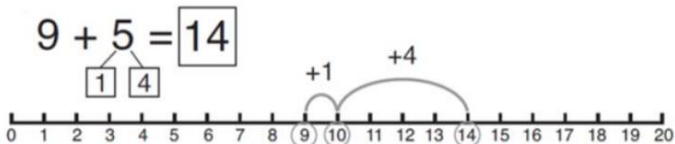


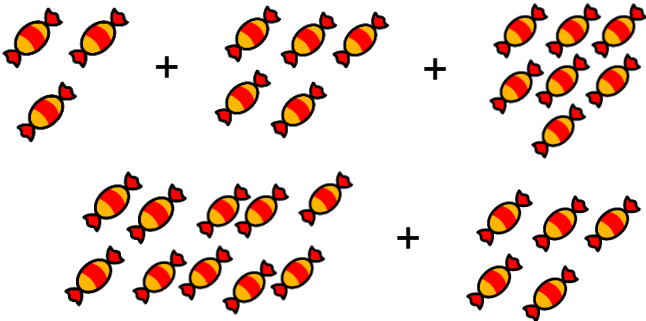
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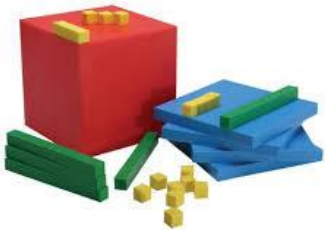
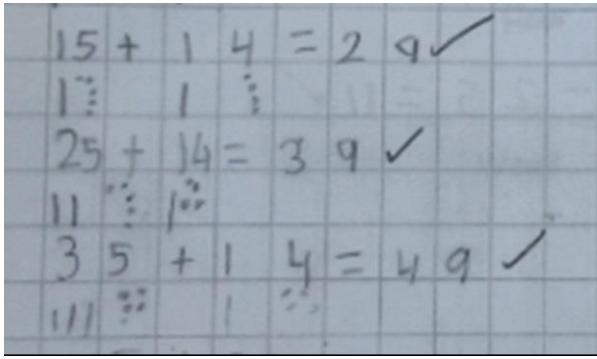
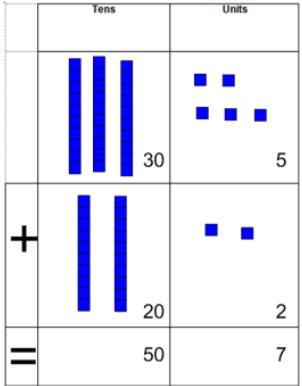
LC2 Overview of Approaches by Year Group

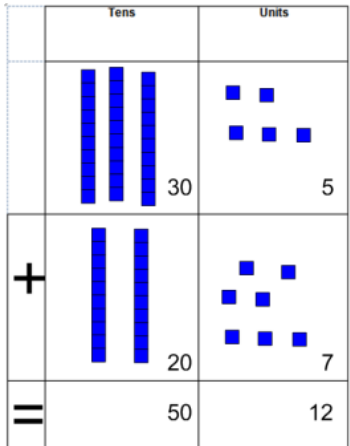
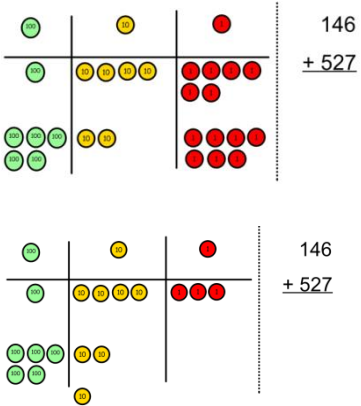
		EYFS	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Use concrete & physical representations. Draw objects. Rhymes and songs. Practical. Mark making.	Combining sets (count all then count on). Partitioning sets and some part part whole. To use a number line practically to add. Use numbers to represent objects, to begin to record.	Combining two parts to make a whole. Part whole model. Starting at the bigger number and counting on. Regrouping to make 10.	Adding three single digits. Partitioning. Column method – no regrouping. Some exchange.	Column method with regrouping (up to 3 digits).	Column method with regrouping. (up to 4 digits).	Column method with regrouping (with more than 4 digits). Column method - decimals - with the same amount of decimal places.	Column method with regrouping. Column method - decimals - with different amounts of decimal places.
Subtraction		Partitioning. Physically take away objects. To use a number line practically to subtract. Use numbers to represent objects, to begin to record.	Taking away ones. Counting back. Find the difference. Part whole model. Make 10.	Counting back; counting on. Find the difference, part whole model, make 10. Column method - no regrouping. Some exchange.	Column method with regrouping (up to 3 digits).	Column method with regrouping (up to 4 digits).	Column method with regrouping (with more than 4 digits). Column method - decimals - with the same amount of decimal places	Column method with regrouping. Column method - decimals - with different amounts of decimal places.
Multiplication		Doubling. Grouping - making equal groups.	Doubling. Equal groups. Counting in multiples. Arrays (with support).	Doubling /halving. Counting in multiples. Repeated addition arrays - showing commutative multiplication. Include x symbol.	Counting in multiples. Repeated addition Arrays - showing commutative multiplication. Grid method with apparatus.	Grid method (as a step towards long multiplication). Column multiplication. (2 and 3 digit multiplied by 1 digit).	Column multiplication (up to 4 digit numbers multiplied by 1 or 2 digits). Decimals.	Column multiplication (multi digit up to 4 digits by a 2 digit number). Decimals.
Division		Doubling/halving. Sharing. Grouping - making equal groups.	Sharing objects. Division as grouping into equal sized groups .	Doubling/halving. Division as grouping and sharing. Division within arrays. Bar model.	Division within arrays. Division with a remainder. Short division (2 digits by 1 digit - concrete and pictorial).	Division within arrays. Division with a remainder / as a fraction. Short division with regrouping (up to 3 digits by 1 digit- concrete and pictorial).	Short division. (up to 4 digits by a 1 digit number - interpret remainders appropriately for the context).	Short division. Long division (up to 4 digits by a 2 digit number - interpret remainders as whole numbers, fractions or round).

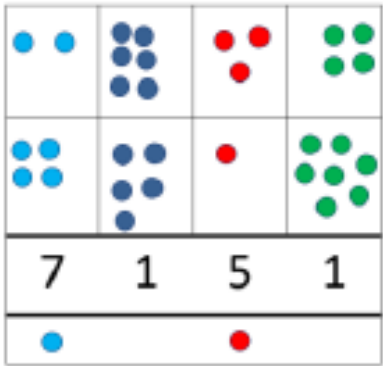
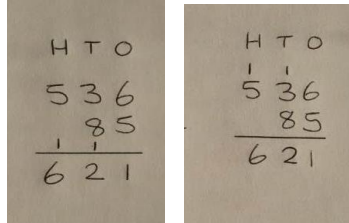
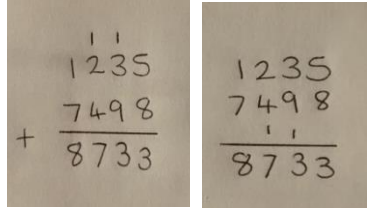
Year	Strategy/Approach	Concrete	Pictorial	Abstract
EYFS	Subitising	Children need to recognise a small group of objects/dots/fingers as the total without counting them	 <p>Use a range of resources e.g. stones before moving to recording</p>	Pupils recognise amounts without counting.
EYFS Y1	Combining two parts to make a whole: part - whole model	  <p>Use cubes to add two numbers together as a group or in a bar.</p>  <p>With lots of real objects</p> 	   <p>Use pictures to add two numbers together as a group or in a bar.</p> 	<p>Consideration is needed when to introduce symbols.</p> $4 + 3 = 7$ $10 = 6 + 4$ $2 + 2 = 3 + 1$  <p>Use the part-part whole diagram as shown above to move into the abstract.</p> <p>Looking for patterns in calculations</p>

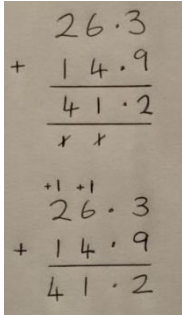
Year	Strategy/Approach	Concrete	Pictorial	Abstract
		<p>Underlying skills:</p> <ul style="list-style-type: none"> • Number recognition 0 -10 and then to 20 • Know that numbers identify how many objects are in a set • Recognise numbers and represent them using objects • Count objects accurately using one to one correspondence matching a number name to each object • Know how to write each number • Being able to count on from numbers other than 0 		<p>Vocabulary:</p> <p>add, more, and make, sum, altogether</p> <p>count on</p> <p>group</p> <p>how many?</p> <p>one more, two more, ten more, Same as</p>
EYFS Y1	Starting at the bigger number and counting on	 <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>	<p>$7 + 4$ In jumps of one.</p>  <p>Children will begin to record their work as number sentences then become more efficient by using facts and not needing to count in ones.</p> <p>$12 + 5 = 17$</p>  <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	<p>$5 + 12 = 17$</p> <p>Place the larger number in your head and count on the smaller number to find your answer. 'Counting' could be automatic from fluency rather than in ones</p>

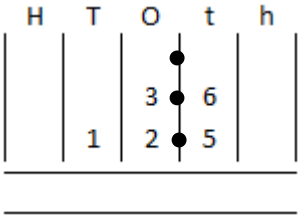
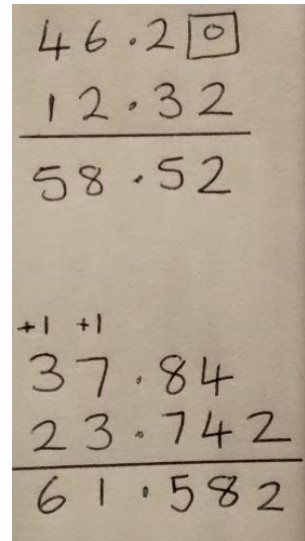
Year	Strategy/Approach	Concrete	Pictorial	Abstract
Y1	Regrouping to make 10	 $6 + 5 = 11$  <p>Start with the bigger number and use the smaller number to make 10</p>	 $3 + 9 =$  <p>Use pictures or a number line. Regroup or partition the smaller number to make 10</p>	$9 + 5 = 14$ <p>If I am at nine, how many more do I need to make 10? How many more do I need to add on now?</p>
Y2	Adding three single digits	$4 + 7 + 6 = 17$  <p>Put 4 and 6 together to make 10. Add on 7.</p>  <p>Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.</p>	 <p>Add together three groups of objects. Draw a picture to recombine the groups to make 10.</p>	$4 + 7 + 6 = 10 + 7 = 17$ <p>Combine the two numbers that make 10 and then add on the remainder.</p> <p>Emphasising the commutative nature/law</p> $4 + 7 + 6 = 17$ $7 + 4 + 6 = 17$ $6 + 7 + 4 = 17$

Year	Strategy/Approach	Concrete	Pictorial	Abstract
Y2	Partitioning	<p>Using Dienes Blocks and Base 10</p> 	<p>This stage will be used by some children.</p>  <p>Use of place value arrow cards can support this.</p>	$64 + 32 = 96$ $4 + 2 = 6$ $60 + 30 = \underline{90}$ $\underline{96}$ <p>OR</p> $60 + 30 = 90$ $4 + 2 = \underline{6}$ $\underline{96}$
Y2	<p>Start of the vertical (column) method</p> <p>No exchange required</p> <p>WHEN CALCULATIONS ARE REPRESENTED VERTICALLY WE START WITH THE LEAST SIGNIFICANT DIGITS FIRST.</p>		<p>At this stage pupils can still link this method with partitioning and work with values of ones and tens.</p> <p>Example:</p> <p>$30 + 20 = 50$ moving to include the language of</p> <p>$3t + 2t = 5t$</p>	$24 + 62 = 88$ <p>No bridging</p>

Year	Strategy/Approach	Concrete	Pictorial	Abstract
Y2	<p>Year 2: TO + TO with exchange</p> <p>(no exchange for expected at Y2 but with exchange for greater depth)</p>	<p>Use a range of apparatus – numicon, straws, beads etc.</p> 	<p>Some pupils will need to make drawings to represent the concrete.</p> <p>App called Number Pieces</p>	<p>Start by partitioning the numbers before moving on to clearly show the exchange below the addition.</p> $\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$
Y3	<p>Column method- regrouping / exchange</p> <p>Year 3: HTO + HTO (no bridging) HTO + HTO (bridging)</p>	<p>Base 10 still needed at concrete stage, then PV counters if appropriate, then abstract. Begin with ones (always).</p> 	<p>Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.</p> <p>The language here needs to refer to the place value column ...</p> <p>For the example on the left the commentary includes ...</p> <p>6 ones and 7 ones equals thirteen ones</p> <p>Four tens add two tens plus the extra one</p>	<div> <p>Expanded method - to be shown as an option to bridge understanding between the pictorial and the abstract</p> $\begin{array}{r} 536 \\ + 85 \\ \hline 11 \text{ (5+6)} \\ 110 \text{ (80+30)} \\ 500 \text{ (500)} \\ \hline 621 \end{array}$ </div> <div> <p>Contracted method</p> $\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$ </div>

Year	Strategy/Approach	Concrete	Pictorial	Abstract
Y4	<p>Year 4: ThHTO + ThHTO (no carrying)</p> <p>ThHTO + ThHTO (carrying)</p> <p>Year 4 – need 4 digits + 4 digits (decimals too).</p> <p>Base 10 still needed at concrete stage, then PV counters, then abstract.</p> <p>Begin with ones (always).</p>	<p>Add up the units and exchange 10 ones for one 10.</p> 	<p>The language here needs to refer to the place value column ...</p> <p>For the example on the left the commentary includes ...</p> <p>4 ones and 7 ones equals eleven ones</p> <p>Three tens add one ten plus the extra ten</p> <p>Six hundreds and five hundreds equal 11 hundreds etc</p>	  <p>Some children may find place value column headings useful.</p> <p>As the children move on, introduce decimals with the same number of decimal places. Money can be used here.</p>

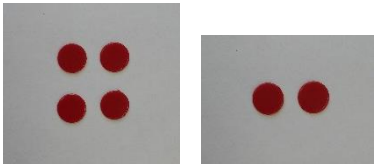



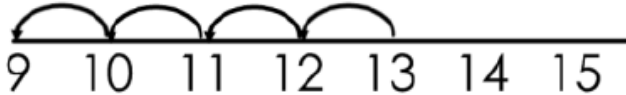
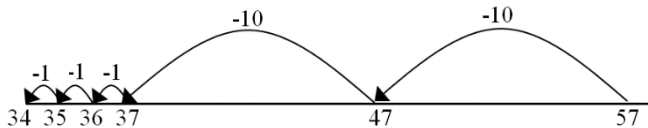
Year	Strategy/Approach	Concrete	Pictorial	Abstract
Y5	Add decimals to 1 and 2 decimal places	<p>Demonstrate how to exchange across the columns so pupils understand the consistency in relationships in place value including decimals.</p> <p>As children move on to decimals, money and decimal place value counters can be used to support learning.</p> <p>Use money as a way in.</p>	<p>Base 10 for decimals</p> <p>Use of place value counters can support pictorial representation of this.</p>	$\begin{array}{r} 72.8 \\ + 54.6 \\ \hline 127.4 \\ 11 \end{array}$ $\begin{array}{r} \pounds 23.59 \\ + \pounds 7.55 \\ \hline \pounds 31.14 \\ 111 \end{array}$  $\begin{array}{r} 26.3 \\ + 14.9 \\ \hline 41.2 \end{array}$ $\begin{array}{r} 26.3 \\ + 14.9 \\ \hline 41.2 \end{array}$ $\begin{array}{r} 23.61 \\ 9.080 \\ 59.770 \\ + 1.300 \\ \hline 93.511 \\ 212 \end{array}$

Year	Strategy/Approach	Concrete	Pictorial	Abstract
Y6	<p>Decimals of differing digits</p> <p>Column method including carrying. Adding decimals of differing digits.</p> <p>HTO</p> $\begin{array}{r} 1 \\ \hline 10 \end{array}$ $\begin{array}{r} 1 \\ \hline 100 \end{array}$	<p>Use Dienes or counters in order to show place value.</p> <p>Vocabulary – add, total, sum, carry, exchange</p> 	<p>It may help pupils to include the place value column headings when writing out calculations.</p>	 <p>And carrying below in other ways.</p>

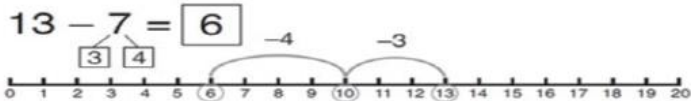
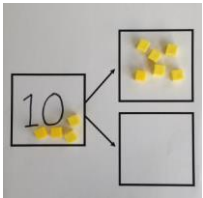
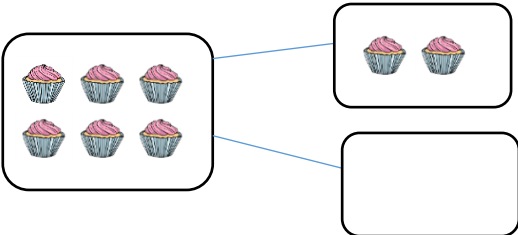
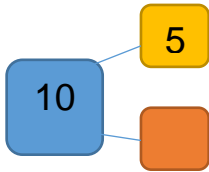
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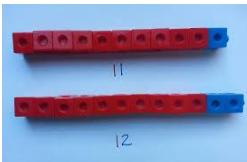
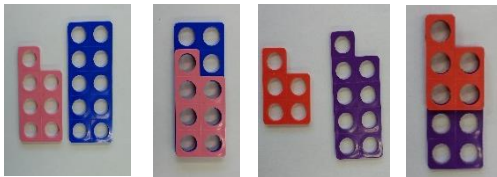
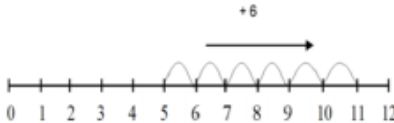
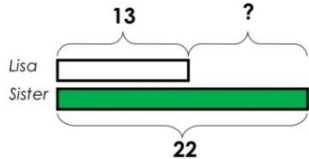
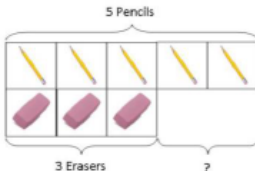
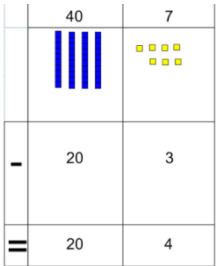
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Subtraction

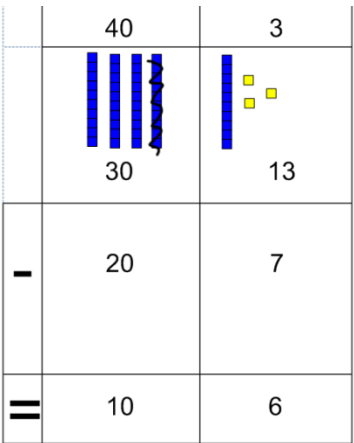
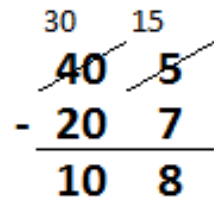
Year	Strategy/Approach	Concrete	Pictorial	Abstract
EYFS	Taking away ones	<p>In a practical context, use physical objects, counters, cubes etc to show how objects can be taken away.</p> <p style="text-align: center;">$4 - 2 = 2$</p> 	<p>Cross out drawn objects to show what has been taken away.</p> 	<p>$18 - 3 = 15$</p> <p>$8 - 2 = 6$</p> <p>(Consideration is needed as to when to use symbols).</p>
EYFS Y1	Counting back	<p>Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.</p> <p style="text-align: center;">$13 - 4 = 9$</p>  <p>Use counters and move them away from the group as you take them away counting backwards as you go.</p> 	<p>Count back on a number line or number track.</p>  <p>Start at the bigger number and count back – become more efficient using number facts rather than counting in ones.</p>  <p>Can progress all the way to counting back using two 2 digit numbers.</p> <p>Do taking away tens, before taking away tens and ones.</p>	<p>Put 13 in your head, count back 4.</p> <p>What number are you at?</p> <p>Use your fingers to help initially; then knowledge and fluency in number facts.</p> <p style="text-align: center;"><u>Y2</u></p> <p>$57 - 23$</p> <p>$57 - 20 - 3$</p> <p>$37 - 3$</p> <p>34</p>

Subtraction

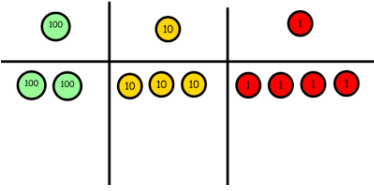
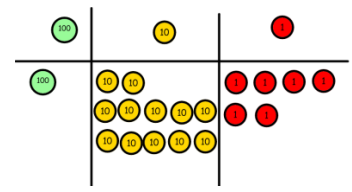
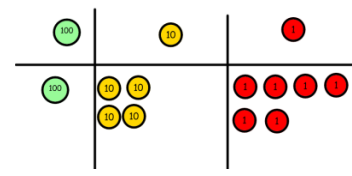
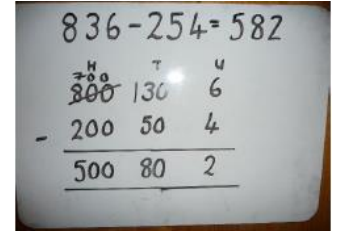
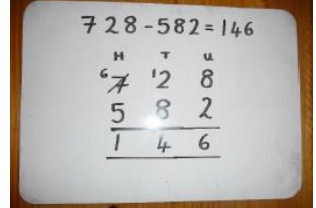
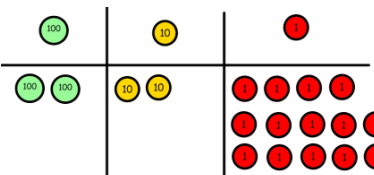
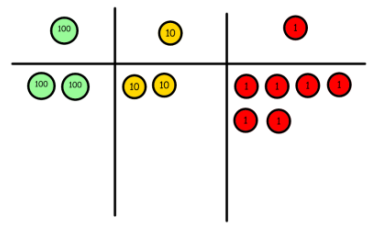
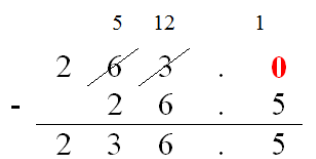
Year	Strategy/Approach	Concrete	Pictorial	Abstract
Y2	Partition numbers to use 10	<p>Calculations below 30</p> <p>Dienes blocks</p> <p>Base 10</p>	 <p>Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.</p>	<p>$16 - 8 =$</p> <p>How many do we take off to reach 10?</p> <p>How many do we have left to take off?</p>
EYFS Y1 Y2	Part Part Whole Model	 <p>Link to addition - use the part part whole model to help explain the inverse between addition and subtraction.</p> <p>If 10 is the whole and 6 is one of the parts. What is the other part?</p> <p style="text-align: center;">$10 - 6 =$</p>	<p>Use a pictorial representation of objects to show the part part whole model.</p>  <p>$6 - 2 = ?$</p> <p>$6 - ? = 2$</p>	 <p>Move to using numbers within the part whole model.</p> <p>$10 - 5 = 5$</p>

Year	Strategy/Approach	Concrete	Pictorial	Abstract
Y2	Find the difference	<p>Compare amounts and objects to find the difference.</p>  <p>Use cubes to build towers or make bars to find the difference</p> 	 <p>Count on to find the difference.</p> <p>Comparison Bar Models</p> <p>Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.</p>  <p>Draw bars to find the difference between 2 numbers.</p>  <p>Use basic bar models with items to find the difference.</p>	<p>Hannah has 23 sandwiches, Helen has 15 sandwiches.</p> <p>Find the difference between the numbers of sandwiches.</p>
Y2 Y3	Column method: No need to regroup or exchange	<p>Start with the ones, or smallest value.</p> <p>Don't write T/O above tens/ones when using the value of the whole number for example '40'.</p> 	Pictorial representation of the concrete.	$\begin{array}{r} 40 \ 7 \\ - 20 \ 3 \\ \hline 20 \ 4 \end{array}$

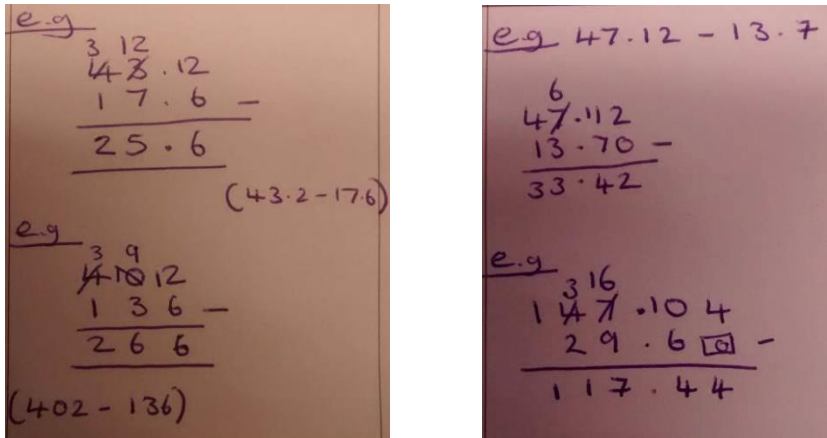
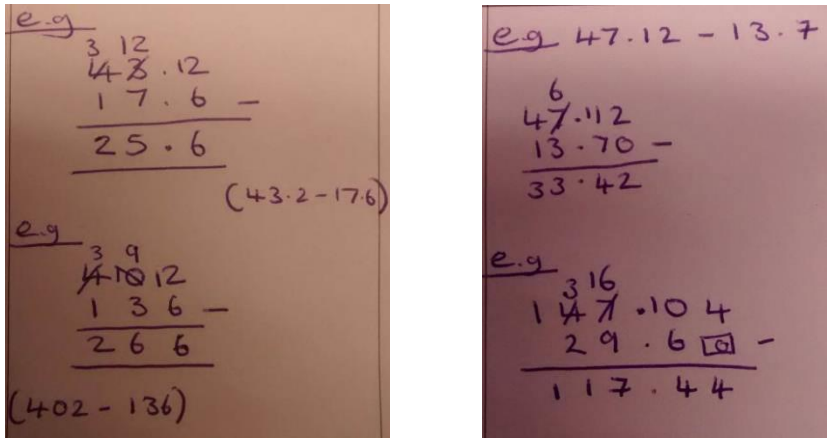
Subtraction



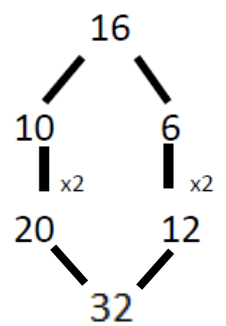
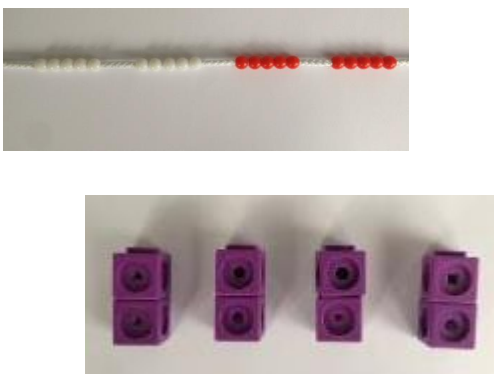
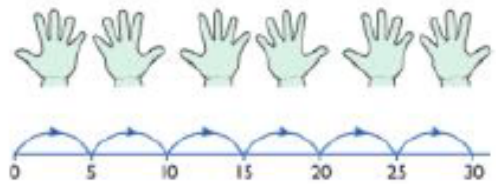
Year	Strategy/Approach	Concrete	Pictorial	Abstract
Y2 (greater depth)	Column method with regrouping or exchange		<p>Say the values of the numbers:</p> <p>Forty and five subtract twenty and seven. Five subtract seven – I don't have enough. So I take ten from forty and add it to the five. I now have thirty and fifteen. I now can take seven from fifteen which is eight. Then twenty from thirty which is ten.</p>	
Y3	Initially with two digit numbers			
Y4	When presenting calculations vertically pupils are taught to start with the least significant digit.			

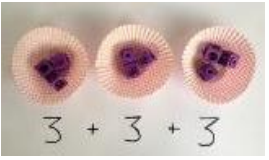


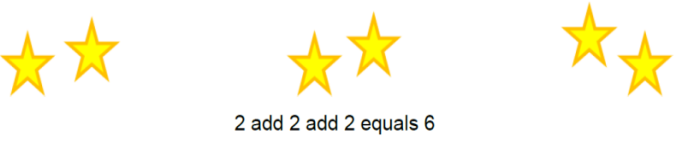
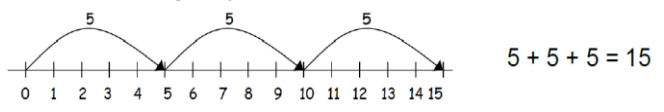




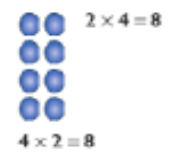
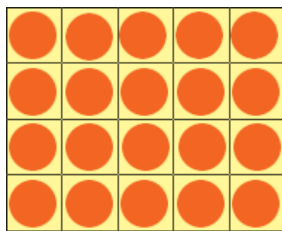

Subtraction

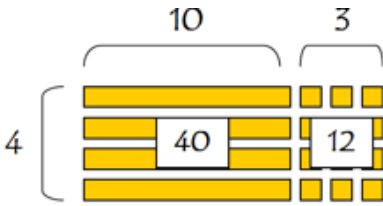
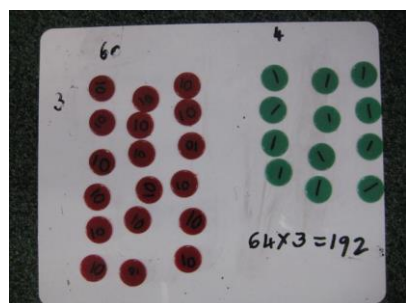
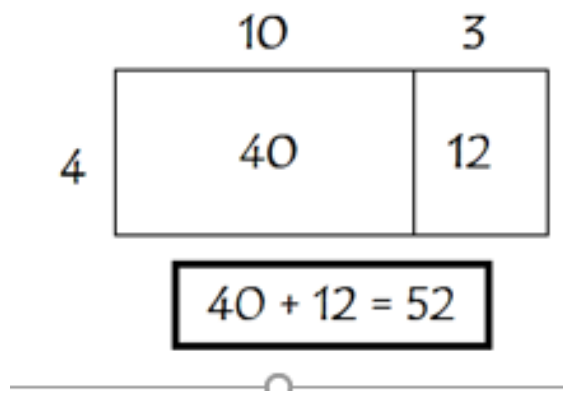
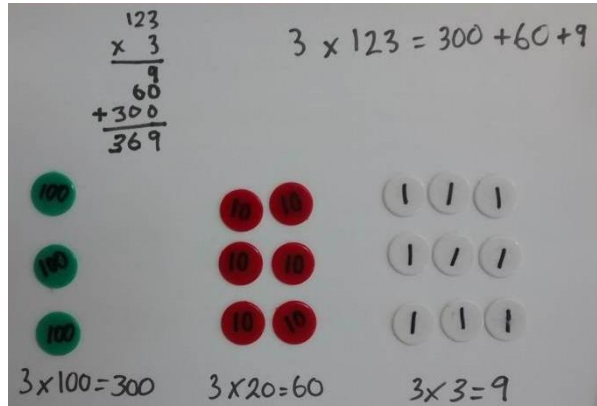
<p>Y3</p>	<p>Column method with regrouping or exchange</p> <p>Larger numbers</p> <p>More than one exchange required</p> <p>Extends to decimals</p> <p>Year 3: HTO - HTO (no exchange)</p> <p>HTO - HTO (exchange)</p>	<div data-bbox="524 165 1043 325"> <p>Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.</p> </div> <div data-bbox="501 373 1043 692"> <p>1 Make the larger number with the place value counters.</p>  <div data-bbox="949 517 1043 644"> <p>Calculations</p> $\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$ </div> </div> <div data-bbox="1106 165 1715 325"> <p>4 Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.</p> </div> <div data-bbox="1173 357 1671 549">  <div data-bbox="1576 389 1671 549"> <p>Calculations</p> $\begin{array}{r} 112 \\ - 88 \\ \hline 24 \end{array}$ </div> </div> <div data-bbox="1106 612 1715 724"> <p>5 Now I can take away eight tens and complete my subtraction.</p> </div> <div data-bbox="1173 772 1671 948">  <div data-bbox="1576 820 1671 948"> <p>Calculations</p> $\begin{array}{r} 112 \\ - 88 \\ \hline 24 \end{array}$ </div> </div>	<div data-bbox="1756 181 2092 421">  </div> <p>Children can start their formal written method by partitioning the number into clear place value columns.</p> <p>Y4</p> <div data-bbox="1778 740 2092 948">  </div> <p>Moving forward the children use a more compact method.</p> <p>This will lead to an understanding of subtracting any number including decimals.</p>
<p>Y4</p>	<p>Year 4: ThHTO - ThHTO (one digit exchanging)</p> <p>ThHTO - ThHTO (more than one digit exchanging)</p> <p>Subtract decimals to 1 and then 2 decimal places.</p>	<div data-bbox="501 724 1043 1059"> <p>2 Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.</p>  <div data-bbox="949 916 1043 1059"> <p>Calculations</p> $\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$ </div> </div> <div data-bbox="501 1123 1043 1442"> <p>3 Now I can subtract my ones.</p>  <div data-bbox="949 1235 1043 1442"> <p>Calculations</p> $\begin{array}{r} 234 \\ - 88 \\ \hline 246 \end{array}$ </div> </div> <div data-bbox="1173 1043 1671 1251"> <p>Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.</p> </div>	<div data-bbox="1778 1299 2092 1458">  </div>

Subtraction

Year	Strategy/Approach	Concrete	Pictorial	Abstract
Y5	Further stages in the method Year 5: Calculations with more than 4 digits including decimals with same number of digits			
Y6	Year 6: Calculations including decimals with different decimal values Ensure vocabulary and language is consistent across year groups – for example, exchange, regroup, steal.			
		Some pupils may find it useful to continue with the place value column headings.	$ \begin{array}{r} 43.7 - 3\frac{1}{4} \longrightarrow 43.70 \\ - 3.25 \\ \hline \end{array} $ $ \begin{array}{r} 4 - 3.75 \longrightarrow 4.00 \\ - 3.75 \\ \hline \end{array} $	

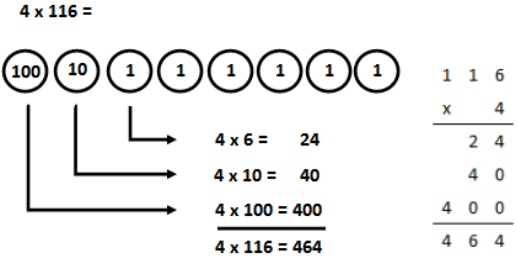
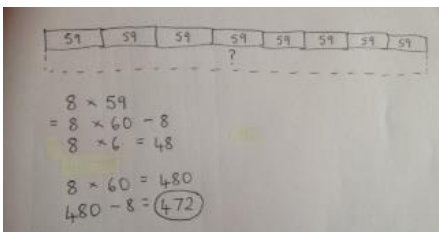
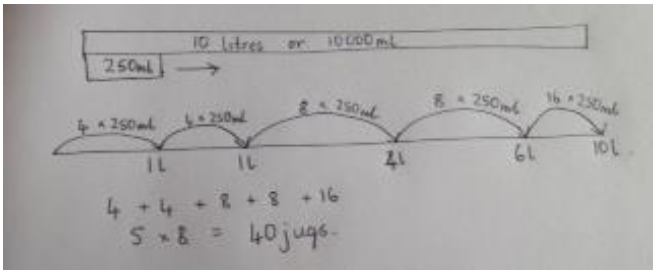
Year	Strategy/Approach	Concrete	Pictorial	Abstract
EYFS Y1 Y2	Doubling	<p>Use practical activities to show how to double a number.</p> 	<p>Draw pictures to show how to double a number.</p> <p>Double 4 is 8</p> 	 <p>Partition a number and then double each part before recombining it back together.</p>
Y1	Counting in multiples	 <p>Count in multiples supported by concrete objects in equal groups. Ensure that pupils do not count in ones.</p>	 <p>Use a number line or pictures to continue support in counting in multiples.</p>	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p>

Year	Strategy/Approach	Concrete	Pictorial	Abstract
Y2	Repeated addition of equal groups	   <div> <p>Use different objects to recognise and add equal groups.</p> </div>	<p>There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?</p>  	<p>Write addition sentences to describe objects and pictures.</p> 
Y2	Arrays – showing commutative multiplication	<p>Create arrays using counters/ cubes to show multiplication sentences.</p>  	<p>Use arrays in different rotations to find commutative multiplication sentences.</p>    <p>Link arrays to area of rectangles.</p> <p> 4×5 or 5×4 </p>	<p>Use an array to write multiplication sentences and reinforce repeated addition.</p>  <p> $5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$ </p>

Year	Strategy/Approach	Concrete	Pictorial	Abstract
Y3	<p>Towards column multiplication</p> <p>2 digits x 1 digit</p>	<p>Children can continue to be supported by place value resources at this stage of multiplication.</p>  		$\begin{array}{r} 13 \\ \times 4 \\ \hline 12 \\ 40 \\ \hline 52 \end{array}$ <p>$13 \times 4 = 52$</p>
Y4	<p>Towards column multiplication</p> <p>2 and 3 digits multiplied by 1 digit</p>			$\begin{array}{r} 123 \\ \times 3 \\ \hline 9 \\ 60 \\ + 300 \\ \hline 369 \end{array}$



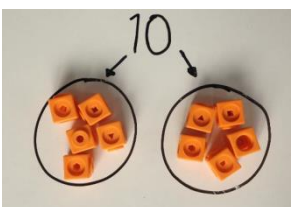
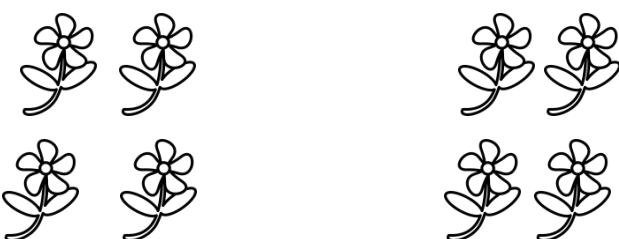
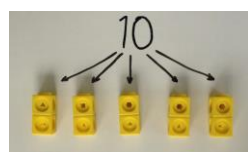
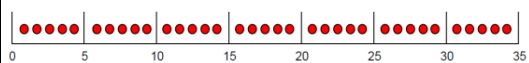
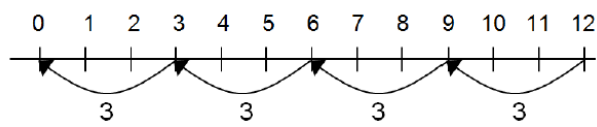
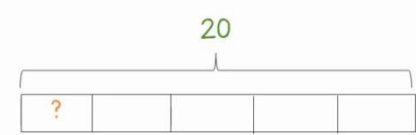
Multiplication

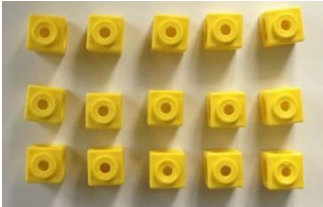
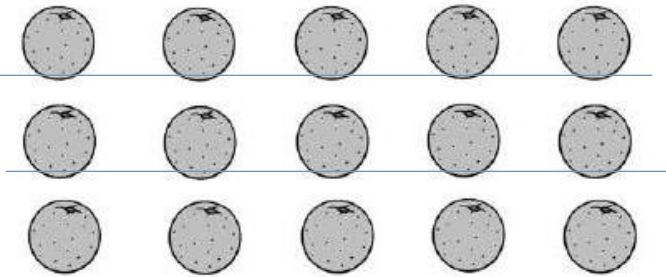
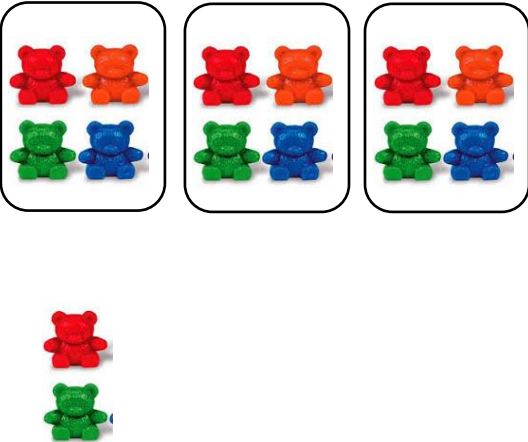
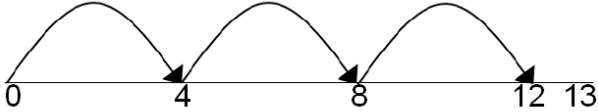

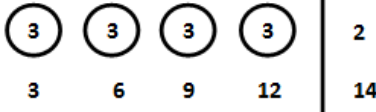
Year	Strategy/Approach	Concrete	Pictorial	Abstract															
Y3	Towards column multiplication	<div><div>8 x 23</div><div></div></div>	33 x 4 (grid method)	Expanded															
Y4	Up to 4 digits by 1 digit	<div><div>8 x 23 = 8 x 10 + 8 x 10 + 8 x 3</div><div>= 80 + 80 + 24</div><div>= 184</div></div>	<div><div>4</div><div><table><tr><td>30</td><td>3</td></tr><tr><td>120</td><td>12</td></tr></table></div><div>=</div><div><table><tr><td>120</td></tr><tr><td>+ 12</td></tr><tr><td>132</td></tr></table></div></div>	30	3	120	12	120	+ 12	132	<div><table><tr><td>33</td></tr><tr><td>x 4</td></tr><tr><td>12 (4 x 3)</td></tr><tr><td>120 (4 x 30)</td></tr><tr><td>132</td></tr></table></div>	33	x 4	12 (4 x 3)	120 (4 x 30)	132			
30	3																		
120	12																		
120																			
+ 12																			
132																			
33																			
x 4																			
12 (4 x 3)																			
120 (4 x 30)																			
132																			
Y5	Towards column multiplication	<div><div>Grid towards long multiplication</div><div><div><div>24 x 16</div><div><table><tr><td>20</td><td>4</td></tr><tr><td>200</td><td>40</td></tr><tr><td>120</td><td>24</td></tr></table></div><div>HTU 240 <div><div>± 144</div><div>384</div></div></div></div></div><div><div>24 x 16 becomes</div><div><div>24</div><div><table><tr><td>16</td></tr><tr><td>144</td></tr><tr><td>240</td></tr><tr><td>384</td></tr></table></div></div></div><div>Answer: 384</div></div> <div><div><table><tr><td>24</td></tr><tr><td>x 16</td></tr><tr><td>144</td></tr><tr><td>240</td></tr><tr><td>384</td></tr></table></div></div>			20	4	200	40	120	24	16	144	240	384	24	x 16	144	240	384
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Year	Strategy/Approach	Concrete	Pictorial	Abstract															
Y5 Y6	Column multiplication	<p>It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.</p> <p>$4 \times 116 =$</p> 	<p>Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.</p>   <p>HTO x TO 372 x 25</p> <table border="1"> <tr> <td>X</td><td>300</td><td>70</td><td>2</td><td></td></tr> <tr> <td>20</td><td>6000</td><td>1400</td><td>40</td><td>7440</td></tr> <tr> <td>5</td><td>1500</td><td>350</td><td>10</td><td>+ 1860</td></tr> </table> <p>= 9300</p>	X	300	70	2		20	6000	1400	40	7440	5	1500	350	10	+ 1860	<p>Start with long multiplication, reminding the children about lining up their numbers clearly in columns.</p> <p>If it helps, children can write out what they are solving next to their answer.</p> $\begin{array}{r} 32 \\ \times 24 \\ \hline 8 \quad (4 \times 2) \\ 120 \quad (4 \times 30) \\ 40 \quad (20 \times 2) \\ 600 \quad (20 \times 30) \\ \hline 768 \end{array}$ $\begin{array}{r} 7 4 \\ \times 6 3 \\ \hline 1 2 \\ 2 1 0 \\ 2 4 0 \\ + 4 2 0 0 \\ \hline 4 6 6 2 \end{array}$ <p>This moves to the more compact method.</p>
X	300	70	2																
20	6000	1400	40	7440															
5	1500	350	10	+ 1860															

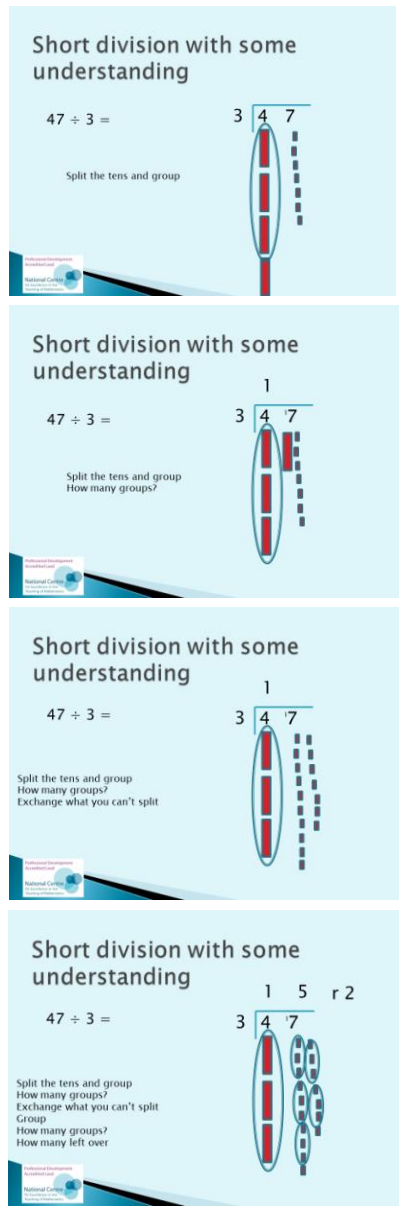
Multiplication

Year	Strategy/Approach	Concrete	Pictorial	Abstract
				$ \begin{array}{r} +1 \\ +1 \quad +2 \\ 1 \quad 4 \quad 7 \\ \times \quad 2 \quad 4 \\ \hline +1 \quad +1 \\ 5 \quad 8 \quad 8 \\ 2 \quad 9 \quad 4 \quad 0 \\ \hline 3 \quad 5 \quad 2 \quad 8 \end{array} $
Y6	<p>Column multiplication</p> <p>Pupils using a column must always start from the right</p>			$ \begin{array}{r} 2 3 1 \\ 1342 \\ \times \quad 18 \\ \hline 13420 \\ 10736 \\ \hline 24156 \\ 1 \end{array} $

Year	Strategy/Approach	Concrete	Pictorial	Abstract
EYFS Y1	Sharing objects into equal groups	   <p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p>  <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $8 \div 2 = 4$ </div>	<p>Share 9 buns between three people.</p> <p style="text-align: center;">$9 \div 3 = 3$</p> <p>Introduce symbols when appropriate</p>
Y1 Y2	<p>Division as grouping</p> <p>Concrete in Y1</p> <p>Moving to abstract in Y2</p>	<p>Divide quantities into equal groups.</p> <p>Use cubes, counters, objects or place value counters to aid understanding.</p>  <p>How many groups of 2 are in 10? Share 10 into groups of 2 is 5 groups $10 \div 2 = 5$</p>  <p>How many groups of 5 are in 35?</p> <p>An early step <u>could</u> include this language leading to 'division'!</p>	<p>Use a number line to show jumps in groups. The number of jumps equals the number of groups.</p>  <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p>  <p style="color: blue; text-align: center;">$20 \div 5 = ?$ $5 \times ? = 20$</p>	<p>$28 \div 7 = 4$</p> <p>Divide 28 into 7 groups. How many are in each group?</p> <p>7 14 21 28 $28 \div 7 = 4$</p> <p>2 4 6 8 10 $10 \div 2 = 5$</p> <p>Consider in Y1 Expected in Y2</p>

Year	Strategy/Approach	Concrete	Pictorial	Abstract
Y2	Division within arrays	 <p>Link division to multiplication by creating an array and thinking about the number sentences that can be created.</p> <p>Eg: $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$</p>	 <p>Use and draw an array and use lines to split the array into groups to make multiplication and division sentences.</p>	<p>Find the inverse of multiplication and division sentences by creating four linking number sentences.</p> <p>$7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$</p>
Y2 (greater depth) Y3	Division with a remainder	<p>$14 \div 3 =$</p> <p>Divide objects into equal groups and see how much is left over</p> 	<p>Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.</p>  <p>Initially draw dots and group them to divide an amount and clearly show a remainder.</p>  <p>Develop to more efficient recording as below.</p> 	<p>Complete written divisions and show the remainder using r.</p> <p>$29 \div 8 = 3 \text{ REMAINDER } 5$</p> <p>↑ ↑ ↑ ↑ dividend divisor quotient remainder</p>

Year	Strategy/Approach	Concrete	Pictorial	Abstract
Y3	<p>Start of chunking which develops to become more efficient</p> <p>Consider introducing the short division method</p>		<p>13 divided by 4</p> $\begin{array}{r} 4 \overline{)13} \\ \underline{4} \\ 9 \\ \underline{8} \\ 1 \end{array}$ <p>"Can I make a group of 4?" "Can I make another group of 4?"</p> $\begin{array}{r} 4 \overline{)13} \\ \underline{4} \\ 9 \\ \underline{8} \\ 1 \end{array}$	<p>It is important to develop efficiency.</p> <p>$21 \div 4 = 5 \text{ r } 1$</p> $\begin{array}{r} 5 \text{ r } 1 \\ 4 \overline{)21} \\ \underline{-4} (1 \times 4) \\ 17 \\ \underline{-4} (1 \times 4) \\ 13 \\ \underline{-4} (1 \times 4) \\ 9 \\ \underline{-4} (1 \times 4) \\ 5 \\ \underline{-4} (1 \times 4) \\ 1 \end{array} \square$ <p>Becoming more efficient and progressing to: $5 \text{ r } 1$</p> $\begin{array}{r} 5 \text{ r } 1 \\ 4 \overline{)21} \\ \underline{-20} (5 \times 4) \\ 1 \end{array}$
Y4	<p>Division with a remainder/as a fraction</p>	<p>Short division becoming more efficient i.e. subtracting in larger chunks.</p>	<p>24 divided by 5</p> $\begin{array}{r} 0 \ 4 \text{ r } 4 \\ 5 \overline{)24} \\ \underline{2} \\ 4 \end{array}$ <p>OR</p> $\begin{array}{r} 0 \ 4 \ 4/5 \\ 5 \overline{)24} \\ \underline{2} \\ 4 \end{array}$	$\begin{array}{r} 12 \\ 6 \overline{)72} \\ \underline{-60} (10 \times) \\ 12 \\ \underline{-12} (2 \times) \\ 0 \end{array}$

Year	Strategy/Approach	Concrete	Pictorial	Abstract
Y3 Y4	Division with regrouping / exchange is introduced in Y3/Y4	<p>Base 10 – dienes</p> <p>Practical modelling of the example in the next column.</p> <p>Consider modelling process with smaller numbers to develop understanding.</p>		<p>98 ÷ 7 becomes</p> $\begin{array}{r} 14 \\ 7 \overline{) 98} \\ \underline{7} \\ 28 \\ \underline{28} \\ 0 \end{array}$ <p>Answer: 14</p> <p>432 ÷ 5 becomes</p> $\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \\ \underline{40} \\ 32 \\ \underline{30} \\ 2 \end{array}$ <p>Answer: 86 remainder 2</p> <p>496 ÷ 11 becomes</p> $\begin{array}{r} 45 \text{ r } 1 \\ 11 \overline{) 496} \\ \underline{44} \\ 56 \\ \underline{55} \\ 1 \end{array}$ <p>Answer: 45 $\frac{1}{11}$</p>
Y5	Dividing up to 4 digits by a single digit using short division			

Year	Strategy/Approach	Concrete	Pictorial	Abstract
	Short division with larger numbers		Some children would benefit from the 'I Know' box.	$ \begin{array}{r} 124 \text{ r}2 \\ 3 \overline{) 1738} \\ \underline{14} \\ 33 \\ \underline{28} \\ 58 \\ \underline{56} \\ 2 \end{array} $
Y5 Y6	Dividing 4 digits by 2 digits using long division		$ \begin{array}{r} 124 \text{ r}2 \\ 14 \overline{) 1738} \\ \underline{14} \\ 33 \\ \underline{28} \\ 58 \\ \underline{56} \\ 2 \end{array} $ <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div> 100×14 20×14 4×14 </div> <div style="border: 1px solid black; padding: 5px;"> <p>I know</p> <p>$2 \times 14 = 28$ $4 \times 14 = 56$</p> <p>$5 \times 14 = 70$</p> <p>$10 \times 14 = 140$</p> <p>$20 \times 14 = 280$</p> <p>$50 \times 14 = 700$</p> <p>$100 \times 14 = 1400$</p> </div> </div>	<p>'I Know' boxes are used to find efficient answers e.g. first subtraction when dividing hundreds are likely to be hundreds/tens.</p>

Division method can be used for higher values using the 'I Know' box. This will aid efficiency which is required as exemplified in the National Curriculum appledix.

<p style="text-align: center;">ADDITION</p> <ol style="list-style-type: none"> 1. No exchange 2. Extra digit in the answer 3. Exchanging ones to tens 4. Exchanging tens to hundreds 5. Exchanging ones to tens and tens to hundreds 6. More than two numbers in calculation 7. As 6 but with different number of digits 8. Decimals up to 2 decimal places (same number of decimal places) 9. Add two or more decimals with a range of decimal places 	<p style="text-align: center;">SUBTRACTION</p> <ol style="list-style-type: none"> 1. No exchange 2. Fewer digits in the answer 3. Exchanging tens for ones 4. Exchanging hundreds for tens 5. Exchanging hundreds to tens and tens to ones 6. As 5 but with different number of digits 7. Decimals up to 2 decimal places (same number of decimal places) 8. Subtract two or more decimals with a range of decimal places
<p style="text-align: center;">SHORT MULTIPLICATION</p> <ol style="list-style-type: none"> 1. TO x O no exchange 2. TO x O extra digit in the answer 3. TO x O with exchange of ones into tens 4. HTO x O no exchange 5. HTO x O with exchange of ones into tens 6. HTO x O with exchange of tens into hundreds 7. HTO x O with exchange of ones into tens and tens into hundreds 8. As 4-7 but with greater number digits x O 9. O.t x O no exchange 10. O.t with exchange of tenths to ones 11. As 9 - 10 but with greater number of digits which may include a range of decimal places x O 	<p style="text-align: center;">SHORT DIVISION</p> <ol style="list-style-type: none"> 1. TO ÷ O no exchange no remainder 2. TO ÷ O no exchange with remainder 3. TO ÷ O with exchange no remainder 4. TO ÷ O with exchange, with remainder 5. Zero in the quotient e.g. $816 \div 4 = \mathbf{204}$ 6. As 1-5 HTO ÷ O 7. As 1-5 greater number of digits ÷ O 8. As 1-5 with a decimal dividend e.g. $7.5 \div 5$ or $0.12 \div 3$ 9. Where the divisor is a two-digit number <p><u>Remainders</u></p> <ol style="list-style-type: none"> 1. Whole number remainder 2. Remainder expressed as a fraction of the divisor 3. Remainder expressed as a simplified fraction 4. Remainder expressed as a decimal